Math 53, Discussion 107

Friday, April 24, 2020 9:37 AM

16.8 (Stoke's Theorem)

16.9 (Divergence Theorem)

**Theorem (Stokes)**

$$\int_C \boldsymbol{F} \cdot d\boldsymbol{r} = \iint_S \nabla \times \boldsymbol{F} \cdot d\boldsymbol{S}$$

Example: compute $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$ with $F = \langle -y^2, x, z^2 \rangle$ and $C$ is the intersection of the plane $y + z = 2$ and $x^2 + y^2 = 1$

Show this reduces to Green's theorem

Interpret this to give meaning of curl

**Theorem (Divergence)**

$$\iint_S \boldsymbol{F} \cdot d\boldsymbol{S} = \iiint_E \nabla \cdot \boldsymbol{F} \, dV$$

Example: compute the flux of $F = \langle z, y, x \rangle$ over the unit sphere.

**Exercises:**

1) Compute $\iint_S \nabla \times \boldsymbol{F} \cdot d\boldsymbol{S}$ **for** $F = \langle 2y \cos z, e^x \sin z, xe^y \rangle$, on the upper half of the sphere centered at the origin with radius 9
2) Compute $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$ for $F = \langle yz, 2xz, e^{xy} \rangle$ $C$ the circle $x^2 + y^2 = 16, z = 5$
3) Compute $\iint_S F \cdot dS$ for $F = \langle exy^2, xe^z, z^3 \rangle$ over the surface bounded by $y^2 + z^2 = 1$ and the $x = -1, x = 2$
4)